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THE BUCKLING OF CURVED TENSION-FIELD GIRDERS

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THE BUCKLING OF CURVED TENSION-FIELD GIRDERS*

By G. Limpert

SUMMARY

The present paper reports on experiments made with a view of determining the buckling load under shear of circular curved tension-field webs. The buckling load of the webs may be expressed with reference to the buckling load of the stiffeners. It is found that within the explored range the buckling load is approximately twice as great as that of the identically stiffened flat wall of equal web depth.

I. INTRODUCTION

The analysis concerns a plate girder comprising a cylindrically curved web and two flanges running lengthwise along the generating axis of the cylinder. The web plate and its stiffeners represent the web (fig. 1).

Under a certain transverse load (buckling - shear stress τ_0) the web plate bulges first. Under further load a share of the transverse load of the web wall is carried in tension. The stiffeners unstressed so far undergo, even if arranged on one side at the sheet wall, a pure compressive stress as a result of deflected tensile stresses (very approximate), whereby the stiffener stress resembles the load of a ring under uniform external compression (reference 1).

With the buckling of the web, i.e., of the stiffeners, together with the web plate, the girder reaches its carrying capacity (buckling load, buckling shear stress τ_k). The shear load under which the curved plate girder buckles, is ascertained from tests, and the stiffener stress computed therefrom.

*"Über die Knickung gekrümmter Zugfeldträger." Luftfahrtforschung, vol. 14, no. 7, July 20, 1937, pp. 356-360.

II. DESCRIPTION OF TEST SPECIMENS

Nine specimens were investigated - of equal length axially (1,500 mm), equal length of stiffeners (500 mm), and equal stiffener spacing (100 mm). All three samples have the same web thickness and the same stiffener section - differing only, respectively, in the curvature and angle at the center (30° , 60° , 90°).

Two metal strips S_1 and S_2 , riveted to both sides of the web plate, form the stiffeners (fig. 2). In order to simulate a linked connection as closely as possible, the stiffeners terminate at two small edge angles G_1 and G_2 placed so close to the clamping rail A substituting for the flange, that the piece of the web in between does not buckle.

Web plate and stiffeners were of sheet brass of tested modulus of elasticity. It is constant up to a tensile stress of $\sigma = 1,000 \text{ kg/cm}^2$ (fig. 3). The wall dimensions can be read from table I.

III. DESCRIPTION OF TEST RIG (fig. 4)

It consists of a four-hinge frame. Rail A is solidly fastened to the test frame, rails B and C provide the parallel guidance of plates D and E. The sides of the specimen walls are clamped to rails A and E. The curved end sections are secured at plates D and E (fig. 4).

The load is applied by a set of pulleys and levers. The introduced transverse load is read on a spring dynamometer, the displacement on a Zeiss dial gage. The angle of the wrinkles is defined by protractor. The anticipated accuracy of the force measurement amounts to $\pm 10 \text{ kg}$. The structural design of the test rig can be seen in figures 5 and 6.

$$\text{kg/cm}^2 \times 14.2235 = \text{lb./sq.in.} \qquad \text{kg} \times 2.20462 = \text{lb.}$$

TABLE I

Sheet No.	Sheet thickness s mm	Angle at center φ°	Radius r cm	Stiffener		$\frac{F_{\text{stiffeners}}}{F_{\text{sheet}}}$	
				section $F_y \text{ cm}^2$	inertia $J_y \text{ cm}^4$		
Series I	1	0.111	31.8	90.0	0.140	6.44×10^{-4}	1.26
	2	.114	60	47.75	.1390	6.17×10^{-4}	1.22
	3	.114	91.2	31.4	.1396	6.17×10^{-4}	1.22
Series II	4	0.220	31.2	90.0	0.174	10.8×10^{-4}	0.79
	5	.225	65.4	43.8	.174	10.8×10^{-4}	.77
	6	.214	89	32.2	.167	10.2×10^{-4}	.78
Series III	7	0.303	30	95.5	0.1927	15.7×10^{-4}	0.64
	8	.307	62.5	45.9	.1932	15.7×10^{-4}	.63
	9	.311	90	31.9	.1897	15.3×10^{-4}	.61

Length l = 1,500 mm E = 1.06×10^6 kg/cm²
Arc length h = 500 mm at s=0.2 and 0.3 mm
Stiffener spacing t_y = 100 mm E = 1.0×10^6 kg/cm²
at s=0.1 mm
(mm×0.03937=in.) (kg/cm²×14.2235=lb./sq.in.)

IV. TEST PROCEDURE

The load is applied in stages and the angle of shearing strain γ , ascertained. The start of buckling of the web plate is observed. As soon as wrinkling is distinctly noticeable, several measurements are made in all panels in wrinkle direction and of the radius of the specimen wall. The directions of the wrinkles are made visible by coloring the wave elevations on both sides of the web plate. The buckling load and the correlated angle of shearing strain are established.

V. RESULTS OF TESTS

The experiments afforded, other than the desired buckling stress in shear, are the bulging stress of the web plate, the angle of shear, and the wrinkling angle. The experimental values are comparable to those obtained from theory and from other available tests.

a) Bulging of Web Plate

The formation of the tension field approximately starts at the buckling loads for curved sheets known from experiments but not simultaneously in the individual panels of the test wall. This is attributable to manufacturing defects - particularly, to unavoidable minor initial bulges.

b) Angle of Shearing Strain, γ

The theoretical as well as the experimental γ values are shown in figure 7. At start of loading the shear modulus, owing to the initial bulges, is not that of the shear-resistant sheet, but approaches that of the tension-field theory. On the developed tension field the rise of the experimental curve resembles approximately the theoretical rise. But on account of the disregarded flexural stiffness of the sheet in the tension-field theory, the theoretical values exceed those of the test, and so much more as the web plate is more curved and thicker. (Specimen sheet No. 1, especially, disclosed numerous initial bulges and consequently, a somewhat different behavior.) The deviation of the test curve signifies that the proportional limit has been reached.*

c) Angle of Wrinkles

The values plotted in figure 8 are averages of measurements made on both sides of the web plate over its entire height and length. The theoretical values are included for

*The test specimens are similar as regards dimensions. They represent only a very limited range of the potential practical cases. A different aspect of the strain curves is therefore entirely feasible.

comparison. The angle of wrinkling under buckling load, given in table II, is obtained by extrapolation (cf. fig. 8).

TABLE II

Angle of Wrinkles for Buckling Load

	Series I			Series II			Series III		
Sheet No.	1	2	3	4	5	6	7	8	9
α° theoretical	35.3	32.3	29.0	34.8	30.7	27.8	34.4	30.7	28.0
α° experimental	38.5	34.9	29.5	38.4	32.4	28.7	36.0	30.7	30.3

d) Buckling Shear Stress and Stiffener Stress

Wall failure is initiated with the buckling of a stiffener - the second to fifth stiffener usually buckling from above. The effect produced by the rigid clamping of the curved end sections, which leads one to suspect an unloading of the stiffeners, therefore appears soon to be canceled. The load then dropped to about 75 percent of the buckling load, although in subsequent load applications, 90 to 95 percent of the buckling load (buckling shear stress τ_k) was obtainable again with very great displacements. The buckling stresses in shear τ_k are appended in table III. Figure 9 shows the failure of specimen sheet No. 1, figure 10 shows a series of failures of sheets No. 3, 6, and 9, which have the same curvature but different thickness and stiffener sections.

In order to extend the range of validity of the experimental data beyond the scope of the investigation, theoretical comparison possibilities were provided for the interpretation of the tests. In the absence of theoretical data on the buckling of such curved tension fields, the comparison of the curved with the flat tension field suggests itself. On flat tension-field webs, it is expedient to express the web buckling load in stiffener stress. One of Wagner's reports (reference 2) gives the ratio of buckling load of the stiffeners to the Euler load of the free bar with pin-jointed ends, as comparative value for the determination of the buckling load of the flat web.

TABLE III

Sheet No.	$\frac{F_y}{s t_y}$	Anglo at center	τ_k	$\frac{\tau_k}{\tau_o}$	σ_y	σ_E	$\frac{\sigma_y}{\sigma_E}$	$\frac{(\sigma_y/\sigma_E)_{\text{curved}}}{(\sigma_y/\sigma_E)_{\text{flat}}}$	
		$\phi = \frac{h}{r}$	$\frac{\text{kg}}{\text{cm}^2}$		$\frac{\text{kg}}{\text{cm}^2}$	$\frac{\text{kg}}{\text{cm}^2}$		v	
Series I	1	1.26	0.556	543	21.6	316	18.6	17	2.43
	2	1.22	1.05	717	17.7	370	18.5	20.0	2.86
	3	1.22	1.59	768	13.87	348	18.45	18.9	2.71
Series II	4	0.79	0.555	522	6.71	462	25.8	17.9	2.56
	5	.77	1.14	678	5.95	521	25.8	20.2	2.89
	6	.78	1.55	590	4.61	404	25.4	15.9	2.27
Series III	7	0.64	0.524	427	3.28	460	34.0	13.5	1.93
	8	.63	1.09	620	3.52	585	32.9	17.8	2.54
	9	.61	1.57	666	3.06	580	33.6	17.25	2.46

The stiffener stress σ_y of the curved tension-field web is therefore computed from the measured buckling-shear stress τ_k and then the ratio

$$v = \frac{(\sigma_y/\sigma_E)_{\text{curved}}}{(\sigma_y/\sigma_E)_{\text{flat}}}$$

computed. (σ_E = Euler stress of free bar with pin-jointed ends). For the quantity $(\sigma_y/\sigma_E)_{\text{flat}}$ the value 7, is the same for all experimental walls, which is valid for the limiting case of very closely spaced stiffeners (reference 2). For the curved sheet the stiffener stress σ_y^* is

*It was not measured in the test: first, because it is difficult on curved webs; secondly, because it merely serves as comparative value for defining the buckling load of the whole wall.

computed from the relation

$$\sigma_y = \tau_k \frac{s t_y}{F_y} \tan \alpha$$

where α , with allowance for τ_0 from figure 8, follows from the report by Wagner and Ballerstedt (reference 1).*

Table III contains the ratio $(\sigma_y/\sigma_E)_{\text{curved}}$ for the individual sheets computed in this manner.

It is to be assumed that the buckling stress in shear τ_k and the ratio v , respectively, depend, aside from the curvature, on other design quantities such as the ratio of sectional area of stiffeners to the correlated section of sheet $F_y/s t_y$, etc. Figure 11 shows v plotted

* In their report the stiffener stress is given as

$$\sigma_y = (\tau - \tau_0) \frac{s t_y}{F_y} \tan \alpha$$

The share of τ_0 still carried herein by the tension field in shear is, however, disregarded in the present report for the stiffener stress determination at the instant of buckling: first, because this share on the flat sheet is unknown; second, because the reduction through τ_0 of the stiffener stress for determining the buckling load of a tension-field web is misleading. For if on a web wall the web plate is so thick that it forms no tension field under transverse load, the stiffeners buckle along with the web plate under a certain fairly low transverse load without being subjected to a compressive stress. If the web plate with unaltered stiffeners is made thinner so as to form a tension field supporting the stiffeners against buckling, the web wall supports a greater transverse load before the stiffeners buckle. The comparison shows that the share τ_0 carried by the tension field in shear, does not lower the stability of the web wall with respect to an ideal tension field. Consequently, to use the stiffener compressive stress established for the ideal tension field to interpret the stability load of the actual tension field, is to overestimate the anticipated tension-field stability, despite the too high mathematical compressive stress.

against the angle at the center ϕ , figure 12 plotted against the ratio $F_y/s t_y$. Although the three test series are unlike in ratio $F_y/s t_y$ as well as σ_k/σ_0 , the experimental v discloses no systematic difference in the test results. On the other hand, the values

$\left(\frac{\sigma_y}{\sigma_E}\right)_{\text{curved}}$ certainly lie above those of the flat tension field, according to figure 11. At $\phi = \approx 60^\circ$, the curvature results in a rise of buckling load of about 2.7 times that of the flat tension-field web.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics

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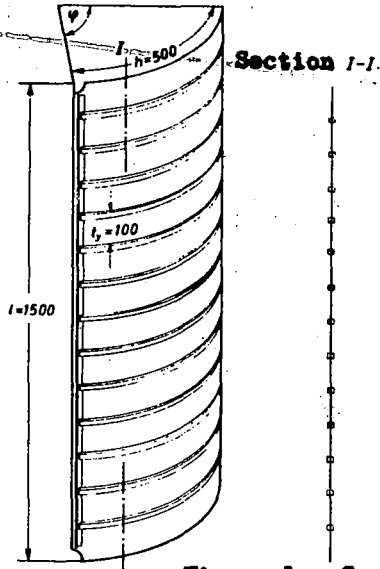


Figure 1.- Specimen sheet.

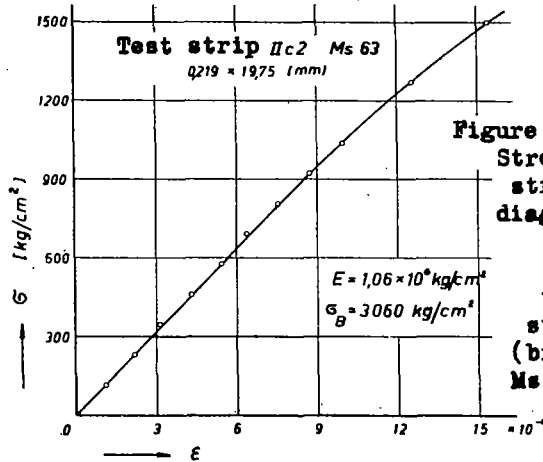


Figure 3.- Stress-strain diagram of a test strip (brass Ms 63)

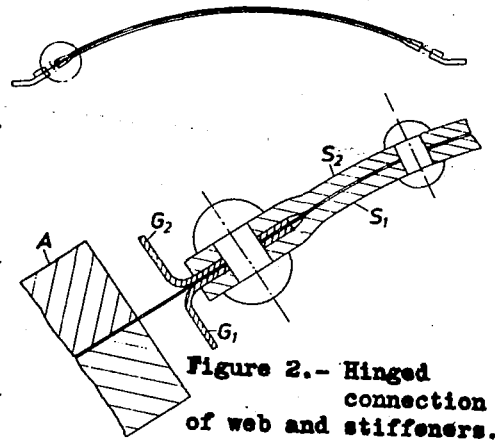


Figure 2.- Hinged connection of web and stiffeners.

Figure 4.- Sketch of test rig.

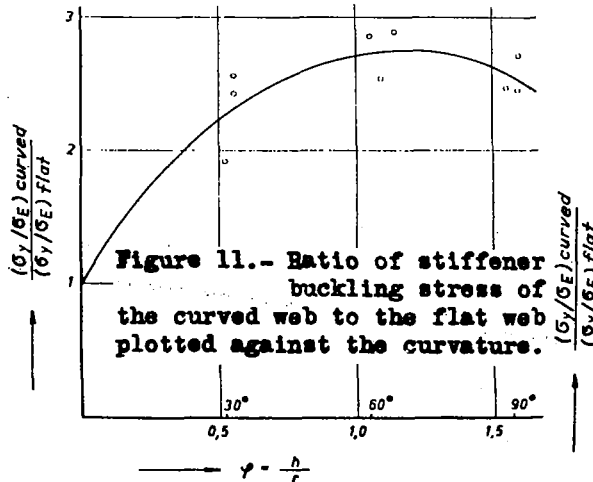
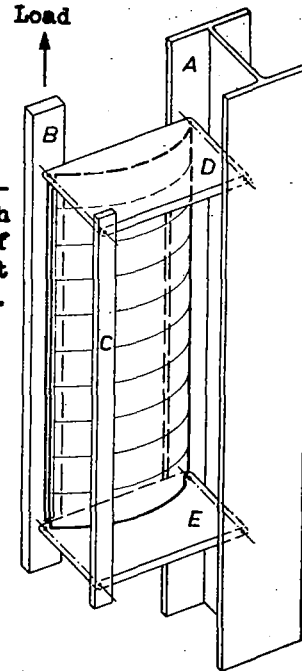


Figure 11.- Ratio of stiffener buckling stress of the curved web to the flat web plotted against the curvature.

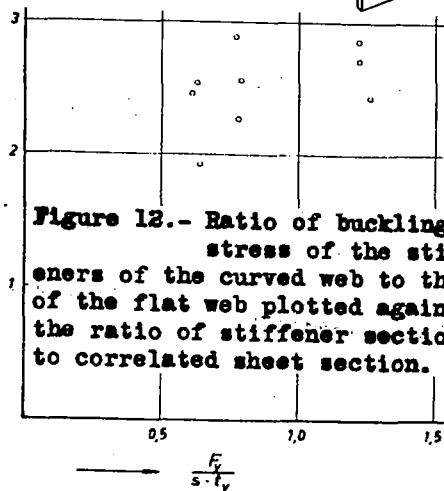


Figure 12.- Ratio of buckling stress of the stiffeners of the curved web to that of the flat web plotted against the ratio of stiffener section to correlated sheet section.

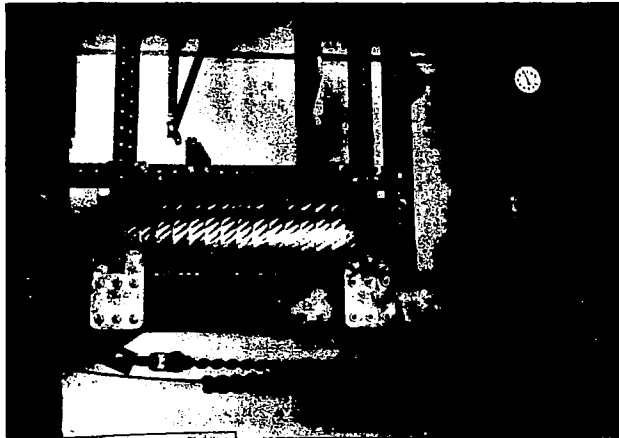


Figure 5.- View of test rig.

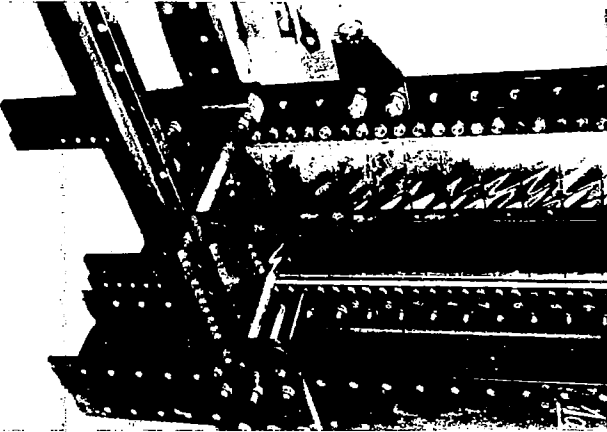


Figure 6.- Partial view of test rig.

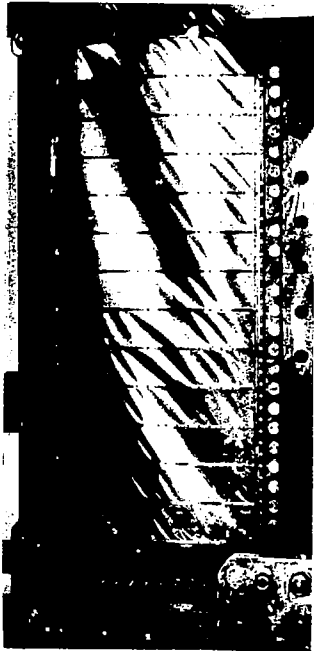
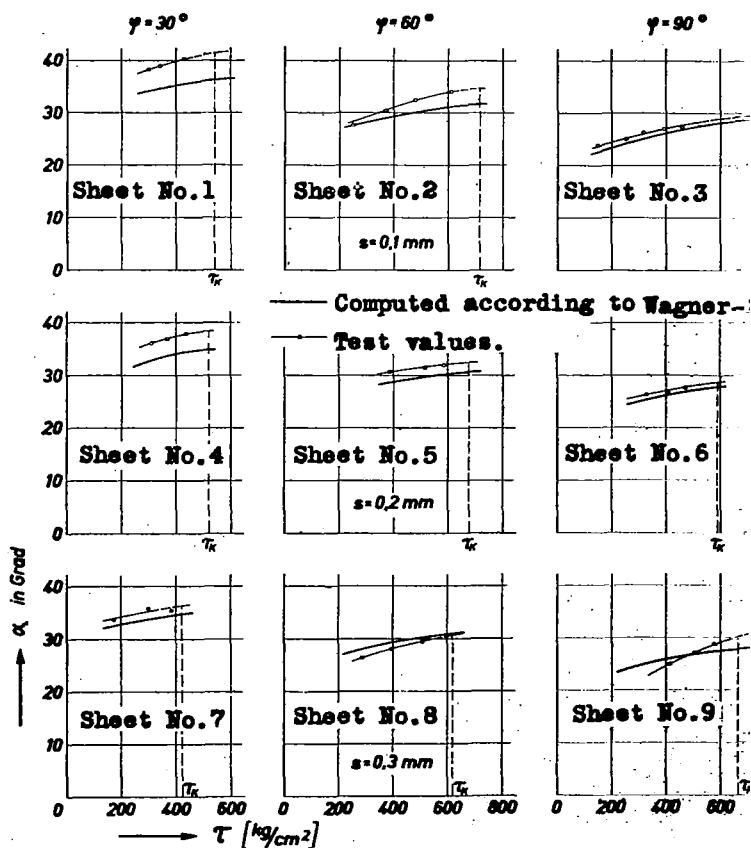
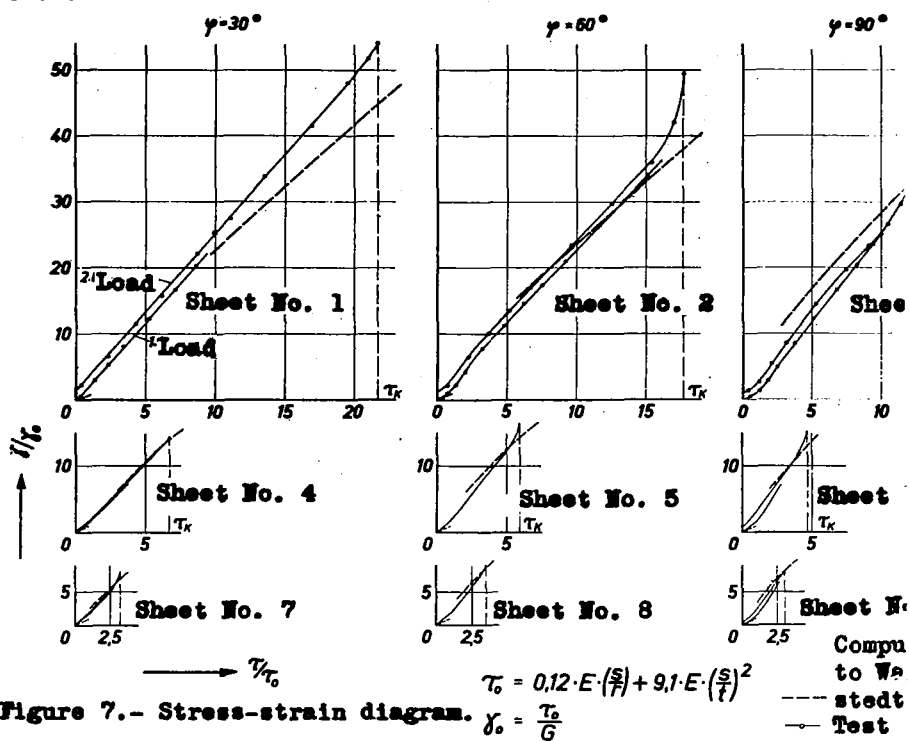


Figure 9.- Test sheet No. 1(failure.)



Figure 10.- Test sheets Nos. 3, 6 and 9(failure)(the dark streak lengthwise in sheet No. 9 is merely the shadow of the rail C).



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